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CPT AND LORENTZ SYMMETRY IN HYDROGEN AND ANTIHYDROGEN^a

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Possibilities for observing signals of CPT and Lorentz violation in the spectroscopy of hydrogen and antihydrogen are considered. We show that transitions between the *c* and *d* hyperfine sublevels in the 1S state can exhibit theoretically detectable effects that would be unsuppressed by powers of the fine-structure constant. This transition may therefore offer some advantages over 1S-2S two-photon spectroscopy.

1 Introduction

The recent production and observation of antihydrogen ($\bar{\text{H}}$)^{1,2} opens new possibilities for precision tests of CPT symmetry. The two-photon 1S-2S transition frequency has been measured to 3.4 parts in 10^{14} in an atomic beam of hydrogen (H)³ and to about one part in 10^{12} in trapped H.⁴ It is hoped that an eventual measurement of the line center to about 1 mHz, corresponding to a resolution of one part in 10^{18} , would be possible.⁵ If such precisions could also be achieved in the spectroscopy of $\bar{\text{H}}$, comparisons of corresponding frequencies in H and $\bar{\text{H}}$ could yield stringent tests of CPT symmetry. Current proposals for $\bar{\text{H}}$ spectroscopy involve both beam and trapped-atom techniques,^{6,7} and are faced with a number of outstanding challenges including the issue of achieving these precisions in *trapped* H and $\bar{\text{H}}$.⁸ We consider the theoretical prospects for placing appropriate bounds on CPT and Lorentz violation in experiments involving the spectroscopy of free or magnetically trapped H and $\bar{\text{H}}$.

All local Lorentz-invariant quantum field theories of point particles, including the standard model and quantum electrodynamics (QED), are invariant under the discrete symmetry CPT.⁹ Attempts to produce a fundamental theory involving gravity often involve string theory and the spontaneous breaking of these symmetries¹⁰ and, in these investigations, the status of CPT symmetry is far less clear. Observable effects of CPT breaking are already known to be small, and so it is reasonable to assume they would be suppressed by at least

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one power of the low-energy scale to Planck scale ratio. Thus, their detection could occur only in extremely sensitive experiments.

In this proceedings, we show that effects of this type can appear in H and \bar{H} spectra at zeroth order in the fine-structure constant. In addition, these effects are theoretically detectable not only in 1S-2S lines but also in hyperfine transitions.

The framework of our analysis is an extension of the standard model and QED¹¹ that includes spontaneous CPT and Lorentz breaking at a more fundamental level. Desirable features of this microscopic theory appear to include energy-momentum conservation, gauge invariance, renormalizability, and microcausality.¹¹ Analyses in the context of this theoretical framework have been done for photon properties,¹¹ neutral-meson experiments,^{10,12,13,14} Penning-trap tests,¹⁵ and baryogenesis.¹⁶

2 Free H and \bar{H}

We first consider the spectra of *free* H and \bar{H} . For H , the electron of mass m_e and charge $q = -|e|$ in the proton Coulomb potential $A^\mu = (|e|/4\pi r, 0)$ is described by a modified Dirac equation arising from the standard-model extension. Taking $iD_\mu \equiv i\partial_\mu - qA_\mu$, the four-component electron field ψ satisfies

$$(i\gamma^\mu D_\mu - m_e - a_\mu^e \gamma^\mu - b_\mu^e \gamma_5 \gamma^\mu - \frac{1}{2} H_{\mu\nu}^e \sigma^{\mu\nu} + i c_{\mu\nu}^e \gamma^\mu D^\nu + i d_{\mu\nu}^e \gamma_5 \gamma^\mu D^\nu) \psi = 0 \quad (1)$$

in units with $\hbar = c = 1$. CPT is violated by the two terms involving the couplings a_μ^e and b_μ^e , while CPT is preserved by the three terms involving $H_{\mu\nu}^e$, $c_{\mu\nu}^e$, and $d_{\mu\nu}^e$. Lorentz invariance is broken by all five couplings, which are assumed to be small.¹¹ Free protons are also described by a modified Dirac equation¹⁵ with corresponding couplings a_μ^p , b_μ^p , $H_{\mu\nu}^p$, $c_{\mu\nu}^p$, and $d_{\mu\nu}^p$. It is possible to eliminate various combinations of these quantities through suitable field redefinitions. In the following, we keep all couplings, thus showing explicitly that these expressions are unobservable.¹¹

Observable effects in the spectra of free H and \bar{H} can be studied using perturbative calculations in the context of relativistic quantum mechanics. In this calculation, the unperturbed hamiltonians and their eigenfunctions are identical for H and \bar{H} . In addition, all perturbative effects from conventional quantum electrodynamics are also the same in both systems. However, the perturbations arising from the CPT- and Lorentz-breaking couplings for the electron in H can differ from those for the positron in \bar{H} . These perturbations are obtained from Eq. (1) by a standard method involving charge conjugation (for \bar{H}) and field redefinitions.¹⁵ Similarly, additional energy perturbations are

generated by the CPT- and Lorentz-breaking couplings for the proton and antiproton, and can be obtained to leading order via relativistic two-fermion techniques.¹⁷

Let the (uncoupled) electronic and nuclear angular momenta be denoted by $J = 1/2$ and $I = 1/2$ respectively, with third components m_J, m_I . Using a perturbative calculation, the energy corrections for the basis states $|m_J, m_I\rangle$ can be found. For protons or antiprotons, we find that the leading-order energy corrections for spin eigenstates have the same mathematical form as those for electrons or positrons, except for the replacement of superscripts e with p on the CPT- and Lorentz-violating couplings.

In H, we find that the leading-order energy shifts in the 1S level are identical to those in the 2S level. Taking m_p for the proton mass, the shifts are

$$\begin{aligned} \Delta E^H(m_J, m_I) \approx & (a_0^e + a_0^p - c_{00}^e m_e - c_{00}^p m_p) \\ & + (-b_3^e + d_{30}^e m_e + H_{12}^e) m_J / |m_J| \\ & + (-b_3^p + d_{30}^p m_p + H_{12}^p) m_I / |m_I|. \end{aligned} \quad (2)$$

Similarly, for $\overline{\text{H}}$, the leading-order energy shifts $\Delta E^{\overline{\text{H}}}$ in the 1S levels are identical to those in the 2S levels, and are given by the expression (2) with the substitutions $a_\mu^e \rightarrow -a_\mu^e$, $d_{\mu\nu}^e \rightarrow -d_{\mu\nu}^e$, $H_{\mu\nu}^e \rightarrow -H_{\mu\nu}^e$; $a_\mu^p \rightarrow -a_\mu^p$, $d_{\mu\nu}^p \rightarrow -d_{\mu\nu}^p$, $H_{\mu\nu}^p \rightarrow -H_{\mu\nu}^p$. We note that because Eq. (2) contains spatial components of the couplings, it would be necessary to take into account the geometry when comparing results from different experiments. For example, measurements taken at different times of the day would be sensitive to different projections of the couplings due to the rotation of the Earth.

The electron and proton spins in H are coupled by the hyperfine interaction, and this is also the case for the positron and antiproton spins in $\overline{\text{H}}$. The total angular momentum F must be considered, and the appropriate basis states become linear combinations $|F, m_F\rangle$ of the $|m_J, m_I\rangle$ states. The allowed two-photon 1S-2S transitions satisfy the selection rules $\Delta F = 0$ and $\Delta m_F = 0$.¹⁸ There are thus four allowed transitions for both H and $\overline{\text{H}}$, those for which the spins remain unchanged. However, no leading-order effects appear in the frequencies of any of these transitions, because according to Eq. (2) the 1S and 2S states with identical spin configurations have identical leading-order energy shifts. Thus in the present theoretical context, there are no signals of Lorentz or CPT violation in free H or in free $\overline{\text{H}}$ at leading-order in 1S-2S spectroscopy. This agrees with results found previously¹⁵ for the Penning trap, showing that observable CPT-violating effects must also involve CT violation and a spin-flip.

To overcome this limitation, one could consider the dominant subleading energy-level shifts involving the CPT- and Lorentz-breaking couplings in free

H and $\overline{\text{H}}$. These would be hard to detect because they arise as relativistic corrections of order α^2 . They do, however, differ for some of the 1S and 2S levels and therefore observable effects could in principle occur. An example is the term proportional to b_3^e in Eq. (1), which produces a frequency shift in the $m_F = 1 \rightarrow m_{F'} = 1$ line relative to the $m_F = 0 \rightarrow m_{F'} = 0$ line (which remains unshifted), given by

$$\delta\nu_{1S-2S}^H \approx -\alpha^2 b_3^e / 8\pi \quad . \quad (3)$$

A similar suppression by a factor at least of order $\alpha^2 \simeq 5 \times 10^{-5}$ would occur in the proton-antiproton corrections. As a result of these suppressions, Penning-trap $g-2$ experiments are likely to be more sensitive to some of the CPT- and Lorentz-violating quantities than experiments involving 1S-2S spectroscopy in free H and $\overline{\text{H}}$. In fact, the estimated attainable bound¹⁵ on b_3^e obtained with existing technology in anomaly-frequency comparisons with electron-positron Penning-trap experiments would suffice to place a bound of $\delta\nu_{1S-2S}^H \lesssim 5 \mu\text{Hz}$ on observable shifts of the 1S-2S frequency in free H from the electron-positron sector. This is beyond the resolution of 1S-2S spectroscopy. For the proton-antiproton quantities in the standard-model extension, experiments have not yet been performed, but bounds attainable would also yield tighter constraints on these parameters than would be possible in 1S-2S spectroscopy.

It is relevant to ask why $g-2$ experiments are potentially more sensitive to observable effects than comparisons of 1S-2S transitions in free H and $\overline{\text{H}}$. This is surprising because the conventional figure of merit for CPT breaking in electron-positron $g-2$ experiments,¹⁹

$$r_g = |g_{e^-} - g_{e^+}| / g_{\text{av}} \lesssim 2 \times 10^{-12} \quad , \quad (4)$$

is six orders of magnitude weaker than the idealized resolution of the 1S-2S line, $\Delta\nu_{1S-2S} / \nu_{1S-2S} \simeq 10^{-18}$. However, the figure of merit r_g in Penning-trap $g-2$ experiments is inappropriate in the present theoretical context.¹⁵ The point is that the experimental sensitivity to CPT- and Lorentz-violating effects is determined by the absolute frequency resolution for unsuppressed transitions. The idealized 1S-2S line-center resolution is about 1 mHz, which would appear to be better than the 1 Hz absolute frequency resolution in $g-2$ measurements. However, $g-2$ experiments are directly sensitive to b_3^e because they involve spin-flip transitions, whereas the 1S-2S transitions in free H or $\overline{\text{H}}$ are sensitive only to the suppressed combination $\alpha^2 b_3^e / 8\pi$. As a result, the bound on b_3^e from electron-positron $g-2$ experiments is thus about two orders of magnitude sharper than that from 1S-2S comparisons.

In addition to the 1S-2S transition, there are certainly others available in H and $\overline{\text{H}}$. The above discussion suggests that transitions between states with

different spin configurations might yield tighter bounds. Such experiments would require external fields to select particular spin states.

3 Trapped H and $\overline{\text{H}}$

We next consider spectroscopy of H or $\overline{\text{H}}$ in the presence of a uniform magnetic field. A way to do this is by confining the particles in a magnetic trap such as an Ioffe-Pritchard trap,²⁰ and imposing an axial bias magnetic field. The situation is directly relevant to proposed experiments.⁷ In the following, we denote each of the 1S and 2S hyperfine Zeeman levels in order of increasing energy in a magnetic field B by $|a\rangle_n$, $|b\rangle_n$, $|c\rangle_n$, $|d\rangle_n$, with $n = 1$ or 2 , for both H and $\overline{\text{H}}$. In the case of H, the four states expressed in terms of the basis states $|m_J, m_I\rangle$ are

$$\begin{aligned} |d\rangle_n &= |\tfrac{1}{2}, \tfrac{1}{2}\rangle \quad , \\ |c\rangle_n &= \sin \theta_n |-\tfrac{1}{2}, \tfrac{1}{2}\rangle + \cos \theta_n |\tfrac{1}{2}, -\tfrac{1}{2}\rangle \quad , \\ |b\rangle_n &= |-\tfrac{1}{2}, -\tfrac{1}{2}\rangle \quad , \\ |a\rangle_n &= \cos \theta_n |-\tfrac{1}{2}, \tfrac{1}{2}\rangle - \sin \theta_n |\tfrac{1}{2}, -\tfrac{1}{2}\rangle \quad . \end{aligned} \quad (5)$$

The mixing angles θ_n are functions of the magnetic field, and are different for the 1S and 2S states:

$$\tan 2\theta_n \approx \frac{(51 \text{ mT})}{n^3 B} \quad . \quad (6)$$

The states $|c\rangle_1$ and $|d\rangle_1$ are low-field seekers, and in principle remain confined near the magnetic-field minimum of the trap. However, a population loss occurs due to spin-exchange collisions $|c\rangle_1 + |c\rangle_1 \rightarrow |b\rangle_1 + |d\rangle_1$ of the $|c\rangle_1$ states over time, so that primarily $|d\rangle_1$ states are confined.

A transition that would seem natural to consider is that between the unmixed-spin states $|d\rangle_1$ and $|d\rangle_2$ because it is field independent for practical values of the magnetic field. The idea would be to compare the frequency ν_d^H for the 1S-2S transition $|d\rangle_1 \rightarrow |d\rangle_2$ in H with the frequency $\nu_d^{\overline{\text{H}}}$ for the corresponding spectroscopic line in $\overline{\text{H}}$. But, in H the spin configurations of the $|d\rangle_1$ and $|d\rangle_2$ states are the same, so any shifts occurring are again suppressed. The same is true for $\overline{\text{H}}$, and so we find

$$\delta\nu_d^H = \delta\nu_d^{\overline{\text{H}}} \simeq 0 \quad (7)$$

at leading order.

Another transition of theoretical interest would be the 1S-2S transition $|c\rangle_1 \rightarrow |c\rangle_2$ in H and the analogous $\overline{\text{H}}$ transition. The point would be to exploit

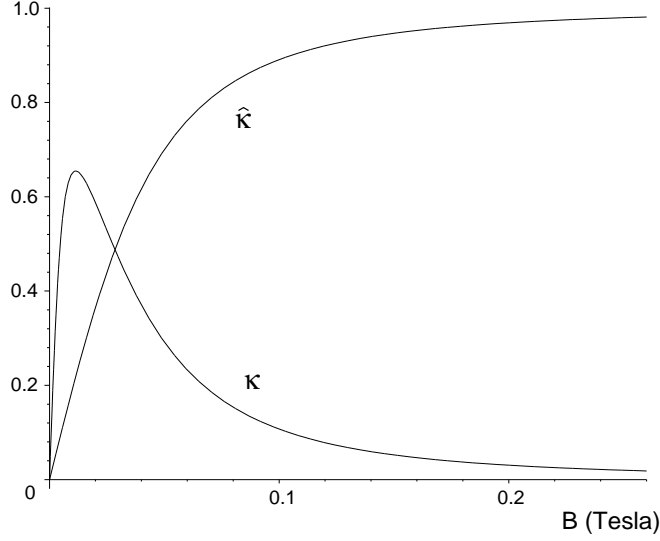


Figure 1: The dimensionless functions κ and $\hat{\kappa}$. For κ , the maximum value of approximately 0.67 occurs at about 0.011 T. The function $\hat{\kappa}$ increases to within about two percent of its asymptotic value (one) as the magnetic field is increased from zero to 0.25 Tesla.

the spin mixing of these states in a nonzero magnetic field. An unsuppressed frequency shift would arise because the hyperfine splitting depends on n , thus producing a spin difference between the 1S and 2S levels in this 1S-2S transition between $|c\rangle_1$ and $|c\rangle_2$:

$$\delta\nu_c^H \approx -\kappa(b_3^e - b_3^p - d_{30}^e m_e + d_{30}^p m_p - H_{12}^e + H_{12}^p)/2\pi . \quad (8)$$

In this expression, κ is a spin-mixing function given by

$$\kappa \equiv \cos 2\theta_2 - \cos 2\theta_1 . \quad (9)$$

This function is always less than one, so to avoid losing sensitivity the optimal situation would involve the largest possible value. This maximum is $\kappa \simeq 0.67$ and occurs at $B \simeq 0.011$ T, as illustrated in Figure 1.

The corresponding 1S-2S shift $\delta\nu_c^{\overline{H}}$ for \overline{H} in the same magnetic field can also be found. Relative to a fixed magnetic field, the hyperfine states in \overline{H} have opposite positron and antiproton spins compared to the electron and proton spins in H . As a result, the expression for $\delta\nu_c^{\overline{H}}$ is identical to that for $\delta\nu_c^H$

in Eq. (8) except that the signs of b_3^e and b_3^p are changed. The frequencies ν_c^H and $\nu_c^{\overline{H}}$ depend on spatial components of Lorentz-violating couplings and would therefore vary diurnally in the comoving Earth frame. Another effect would be an instantaneous difference

$$\Delta\nu_{1S-2S,c} \equiv \nu_c^H - \nu_c^{\overline{H}} \approx -\kappa(b_3^e - b_3^p)/\pi \quad (10)$$

for measurements made in the same magnetic trapping fields.

The transition $|c\rangle_1 \rightarrow |c\rangle_2$ when compared with the transition $|d\rangle_1 \rightarrow |d\rangle_2$ is theoretically more sensitive to CPT and Lorentz violation by a factor of order $4/\alpha^2 \simeq 10^5$. However, the 1S-2S transition $|c\rangle_1 \rightarrow |c\rangle_2$ in H and \overline{H} depends on the magnetic field, and the resultant Zeeman broadening due to the inhomogeneous trapping fields would have to be overcome. Even at a temperature of $100\mu\text{K}$, the transition in both H and \overline{H} would be broadened to over 1 MHz for $B \simeq 10$ mT. This would severely hinder the experimental attainment of resolutions on the order of the natural line width.

Figure 2 illustrates one case for the conventional and perturbed frequencies in the four 1S-2S transitions. In this figure, $b_3^p > 0$ and all the other couplings are zero.

4 Hyperfine Transitions

We now consider the possibilities for spectroscopy of the hyperfine 1S levels. Motivated by the fact that transitions between the $F = 0$ and $F' = 1$ hyperfine states can be measured with accuracies better than 1 mHz in a hydrogen maser,²¹ hyperfine transitions in masers and in trapped H and \overline{H} are worth considering for tests of CPT and Lorentz symmetry.

The energy levels of all four hyperfine states in the ground state of hydrogen are shifted due to CPT- and Lorentz-violating effects. All the shifts contain an identical contribution $a_0^e + a_0^p - c_{00}^e m_e - c_{00}^p m_p$ that leaves energy differences unaffected. The remaining spin-dependent terms are

$$\begin{aligned} \Delta E_a^H &\simeq \hat{\kappa}(b_3^e - b_3^p - d_{30}^e m_e + d_{30}^p m_p - H_{12}^e + H_{12}^p) \quad , \\ \Delta E_b^H &\simeq b_3^e + b_3^p - d_{30}^e m_e - d_{30}^p m_p - H_{12}^e - H_{12}^p \quad , \\ \Delta E_c^H &\simeq -\Delta E_a^H \quad , \quad \Delta E_d^H \simeq -\Delta E_b^H \quad , \end{aligned} \quad (11)$$

where $\hat{\kappa} \equiv \cos 2\theta_1$. If there is no magnetic field, then $\hat{\kappa} = 0$ and the energies of $|a\rangle_1$ and $|c\rangle_1$ are unshifted. However, equal and opposite energy shifts occur for $|b\rangle_1$ and $|d\rangle_1$. The degeneracy of the three $F = 1$ ground-state hyperfine

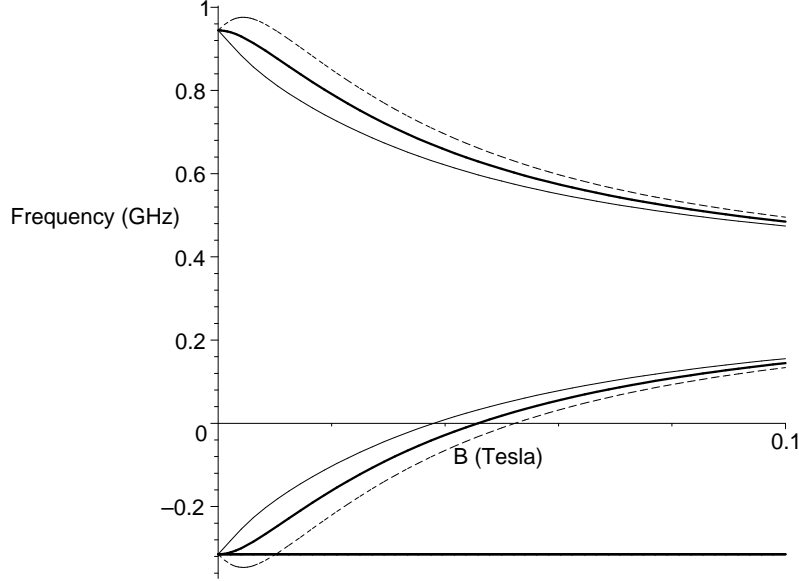


Figure 2: Conventional and perturbed frequencies for the 1S-2S transition as a function of magnetic field. The vertical scale is the shift in the usual Bohr-model 1S-2S frequency of about 2.5×10^{15} Hz. The bold lines are for the conventional frequencies, the fainter solid line is for the perturbed hydrogen transition frequencies, and the dashed line is for the perturbed antihydrogen frequencies. We have taken $b_3^p > 0$, with all other couplings zero. The upper set of three lines represents the $|a\rangle_1 \rightarrow |a\rangle_2$ transition, and the lower set the $|c\rangle_1 \rightarrow |c\rangle_2$ case. The single straight line is for the $|b\rangle_1 \rightarrow |b\rangle_2$ and $|d\rangle_1 \rightarrow |d\rangle_2$ cases, showing how these transitions are field independent and unperturbed by the b_3^p coupling.

levels is therefore removed even for $B = 0$.^b For instance, the $|d\rangle_1 \rightarrow |a\rangle_1$ and $|b\rangle_1 \rightarrow |a\rangle_1$ transitions differ in their frequencies by the unsuppressed and diurnally varying quantity

$$|\Delta\nu_{d-b}^H| \approx |b_3^e + b_3^p - d_{30}^e m_e - d_{30}^p m_p - H_{12}^e - H_{12}^p|/\pi \quad . \quad (12)$$

In the presence of a magnetic field, all four hyperfine Zeeman energy levels are shifted. For the $|a\rangle_1$ and $|c\rangle_1$ states, the spin-mixing function $\hat{\kappa}$ controls the

^bNo conflict with Kramer's theorem occurs in the breaking of the $|b\rangle$ - $|d\rangle$ degeneracy at zero field, because the Lorentz-violating coefficients in Eq. (11) break time-reversal symmetry. A possible method of detecting the splitting might involve looking directly for a difference frequency.

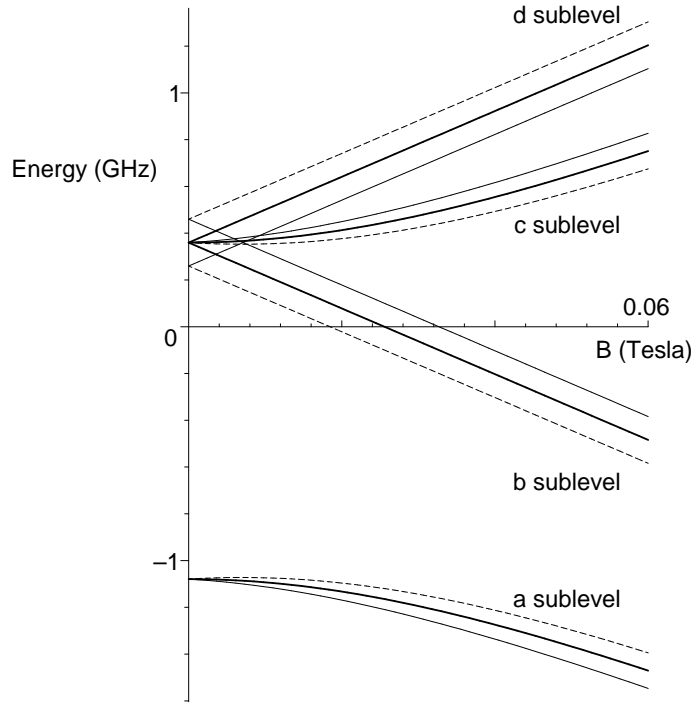


Figure 3: Hyperfine levels for 1S states versus magnetic field. The vertical axis represents the shift in energy (in frequency units) relative to the usual Bohr-model $n = 1$ energy of -13.6 eV. The bold solid line is for the unperturbed case, the finer solid line and the dashed lines are for hydrogen and antihydrogen respectively. We have taken $b_3^p > 0$ and all other couplings zero.

shifts. As B increases from zero, $\hat{\kappa}$ increases, attaining $\hat{\kappa} \simeq 1$ when $B \simeq 0.3$ T. The function $\hat{\kappa}$ is illustrated in Fig. 1. The shifts in the energy levels as given in Eq. (11) are partially illustrated in Figure 3.

The usual H maser employs a small ($B \lesssim 10^{-6}$ T) magnetic field and works with the field-independent σ transition $|c\rangle_1 \rightarrow |a\rangle_1$. The leading-order effects from CPT and Lorentz violation in high-precision measurements of this line $|c\rangle_1 \rightarrow |a\rangle_1$ are suppressed, because for this situation $\hat{\kappa} \lesssim 10^{-4}$. However, a shift $\Delta\nu_{d-b}^H$ does occur in the frequency difference between the field-dependent transitions $|d\rangle_1 \rightarrow |a\rangle_1$ and $|b\rangle_1 \rightarrow |a\rangle_1$ relative to the conventional value, and the associated diurnal variations would provide an unsuppressed signal of CPT and Lorentz violation. The resolution of this difference would be reduced by

broadening due to field inhomogeneities. In addition, it would be necessary to distinguish it from possible backgrounds due to residual Zeeman splittings.

The direct comparison of transitions between hyperfine Zeeman levels in H and $\overline{\text{H}}$ could address the issue of background splittings. Moreover, the magnetic-field dependence of the frequency could be eliminated to first order by working at an appropriate value of the field. One option might be to consider high-resolution spectroscopy at the field-independent transition point $B \simeq 0.65$ T on the $|d\rangle_1 \rightarrow |c\rangle_1$ transition in trapped H and $\overline{\text{H}}$. Experimental hurdles would include Doppler broadening and potentially larger field inhomogeneities due to the relatively high bias field. Obtaining frequency resolutions of order 1 mHz would be a challenge, requiring cooling to temperatures of order 100 μK with a good signal-to-noise ratio and a stiff box shape for the trapping potential.

At this bias-field strength, the electron and proton spins in state $|c\rangle_1$ interact more strongly with the field than with each other and are highly polarized with $m_J = 1/2$ and $m_I = -1/2$. Thus, the transition $|d\rangle_1 \rightarrow |c\rangle_1$ is in essence a proton spin-flip. For this transition, we obtain frequency shifts

$$\begin{aligned}\delta\nu_{c \rightarrow d}^H &\approx (-b_3^p + d_{30}^p m_p + H_{12}^p)/\pi \quad , \\ \delta\nu_{c \rightarrow d}^{\overline{\text{H}}} &\approx (b_3^p + d_{30}^p m_p + H_{12}^p)/\pi\end{aligned}\tag{13}$$

for H and $\overline{\text{H}}$ respectively. One way to detect such terms would be to search for diurnal variations in the frequencies $\nu_{c \rightarrow d}^H$ and $\nu_{c \rightarrow d}^{\overline{\text{H}}}$. Another possibility would be to consider their instantaneous difference,

$$\Delta\nu_{c \rightarrow d} \equiv \nu_{c \rightarrow d}^H - \nu_{c \rightarrow d}^{\overline{\text{H}}} \approx -2b_3^p/\pi \quad .\tag{14}$$

This difference could provide a direct, clean, and sharp test of the CPT-violating coupling b_3^p for the proton.

We can introduce dimensionless figures of merit appropriate for experiments investigating various direct and diurnal-variation signals. This is done in analogy with definitions made for similar tests in Penning traps.¹⁵ As an example, a figure of merit for the signal in Eq. (14) could be chosen as

$$\begin{aligned}r_{rf,c \rightarrow d}^H &\equiv |(\mathcal{E}_{1,d}^H - \mathcal{E}_{1,c}^H) - (\mathcal{E}_{1,d}^{\overline{\text{H}}} - \mathcal{E}_{1,c}^{\overline{\text{H}}})|/\mathcal{E}_{1,\text{av}}^H \\ &\approx 2\pi|\Delta\nu_{c \rightarrow d}|/m_H \quad .\end{aligned}\tag{15}$$

Here, $\mathcal{E}_{1,d}^H$, $\mathcal{E}_{1,c}^H$ and the corresponding quantities for $\overline{\text{H}}$ are relativistic energies in ground-state hyperfine levels, and m_H is the atomic mass of H. If, for example, a frequency resolution of 1 mHz were attained, this would correspond to an upper bound of about $r_{rf,c \rightarrow d}^H \lesssim 5 \times 10^{-27}$. The CPT- and Lorentz-violating

coupling b_3^p would then be limited to $|b_3^p| \lesssim 10^{-18}$ eV. This is about three orders of magnitude better than estimated attainable bounds¹⁵ from $g-2$ experiments in Penning traps and more than four orders of magnitude better than the limit attainable from 1S-2S transitions. We also note that the frequency resolution of high-precision clock-comparison experiments, which can also bound Lorentz violation, lies below $1 \mu\text{Hz}$.²² In these experiments, leading-order bounds are obtained on b_3^p in combination with other couplings.²³ Since the nuclei involved are relatively complex, the theoretical analysis prevents b_3^p from being isolated.

The experiments discussed here are sensitive only to spatial components of CPT-violating couplings. A boost would be needed to be sensitive to timelike components such as b_0^e , and would also enhance CPT- and Lorentz-violating effects.¹⁴ This would be an advantage of the proposed experiments⁶ measuring the fine structure and Lamb shift with a relativistic beam of $\bar{\text{H}}$. Although they would probably have poorer resolutions than the others discussed here, constraints on b_0^e and b_0^p may be possible.

In conclusion, we have shown that 1S-2S transitions involving the mixed-spin $|c\rangle$ states as well as the spin-flip $|d\rangle_1 \rightarrow |c\rangle_1$ hyperfine transition could give rise to signals of Lorentz and CPT violation in magnetically confined H or $\bar{\text{H}}$ atoms. These signals would not be suppressed by powers of the fine-structure constant. They would indicate observable and qualitatively new physics originating at the Planck scale.

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